Journal of Sound and Vibration (2000) **232**(4), 834–837
doi:10.1006/jsvi.1999.2765, available online at http://www.idealibrary.com on IDEAL®

A NOTE ON "THE FINITE RESIDUAL MOTION OF A DAMPED THREE-DEGREE-OF-FREEDOM VIBRATING SYSTEM"

P. HAGEDORN

Institut für Mechanik, Darmstadt University of Technology, Hochschulstrasse 6, *D*-64289 *Darmstadt, Germany*

(*Received* 23 *August* 1999)

Recently, in a letter to the editor of this journal Wilms and Pinkney discussed a damped linear three-degree-of-freedom system with two different types of damping matrices (see reference [1]). They examined the question whether or not undamped motions are possible in their systems. Similar two-degree-of-freedom examples had been examined earlier in this journal, the references being given in reference [1].

It seems to have escaped the authors' attention that the question examined in these publications is the one of *pervasiveness* of damping in a linear system. This is an important albeit rather elementary concept which is not new but unfortunately not always discussed in textbooks. It is, for example, introduced by Meirovitch in his now classical book [2] and also presented in many other vibration texts, for example in references [3–5]. Since it does not seem to be sufficiently well known, here is a short discussion of this property.

Consider the n-degree-of-freedom system

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \tag{1}$$

with the real n-dimensional vector $\mathbf{q}$ of generalized co-ordinates and the real $n \times n$ matrices $\mathbf{M}^{\mathrm{T}} = \mathbf{M} > 0$, $\mathbf{D}^{\mathrm{T}} \geqslant 0$, $\mathbf{K}^{\mathrm{T}} = \mathbf{K} > 0$. In the particular case $\mathbf{D} > 0$ (positive definite rather than semi-definite) the damping is sometimes called *complete*. For $\mathbf{D} > 0$, all the free motions are damped and there is no "finite residual motion". However, $\mathbf{D} > 0$ is only a *sufficient*, but *not a necessary condition* for all motions being damped.

If the matrices **M**, **D** and **K** are such that all the motions are damped, i.e. there is no "finite residual motion", then the system is said to have *pervasive damping*. As the example given in reference [1] shows very clearly, the damping matrix **D** does not need to be definite in order for the damping to be pervasive. Pervasiveness of the damping is not determined by the damping matrix **D** only, but in general depends on all three matrices **M**, **D** and **K**.

The concept of pervasiveness of damping is related to the concept of *controllability of a linear system*. As shown in reference [4], system (1) is pervasively damped if and only if the system

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{D}\mathbf{u} \tag{2}$$

is completely controllable via the control **u** (**u** is an n-dimensional vector). Correspondingly, all the matrix criteria for controllability of linear systems can be used to check the pervasiveness of damping. The most obvious criterion, although not necessarily the most practical one, is

$$\operatorname{rank}(-\omega^2\mathbf{M} + \mathbf{K} - \mathbf{D}) = n \tag{3}$$

0022-460X/00/190834 + 04 $35.00/0

for each ω (circular eigenfrequency of the undamped system). Equation (3) simply indicates that none of the eigenvalues of the undamped system is at the same time also an eigenvalue of the damped system.

Of course, other criteria for controllability can also be used to check the pervasiveness of damping. For example, a criterion for pervasive damping, not involving the circular eigenfrequencies of the undamped system results from Kalman's criterion

$$\mathrm{rank}(\mathbf{D}(\mathbf{M}^{-1}\mathbf{K})(\mathbf{M}^{-1}\mathbf{D})(\mathbf{M}^{-1}\mathbf{K})^2(\mathbf{M}^{-1}\mathbf{D})\dots(\mathbf{M}^{-1}\mathbf{K})^{n-1}(\mathbf{M}^{-1}\mathbf{D})) = n \tag{4}$$

(see e.g. [reference 4, Theorem 6.9, p. 165]; the German expression "*durchdringende Dämpfung*" is used for *pervasive damping*).

The author feels that the concept of pervasive damping is extremely relevant to engineering vibrations, since engineers may wish to damp all the free vibrations of a system, introducing a few dampers or dashpots only. In this case, the damping matrix **D** will not be positive definite, but damping should still be pervasive. The concept is therefore used exhaustively in all vibration courses taught by the author. It can and should of course also be generalized to continuous and also to non-linear systems and this is regularly done. For a discussion of the history of the concept of pervasive damping see [reference 5, p. 156].

REFERENCES

1. E. V. Wilms and R. B. Pinkney 1999 *Journal of Sound and Vibration* **224**, 937–940. The finite residual motion of a damped three-degree-of-freedom vibrating systems.
2. L. Meirovitch 1994 *McGraw-Hill Classic Textbook Reissue*. New York: McGraw-Hill. Methods of analytical dynamics.
3. P. Hagedorn and S. Otterbein 1987 *Technische Schwingungslehre*. Berlin: Springer-Verlag.
4. P. C. Müller 1977 *Stabilität und Matrizen*. Berlin: Springer-Verlag.
5. K. Huseyin 1978 *Vibrations and Stability of Multiple Parameter Systems*. Leiden: Noordhoff.

AUTHORS' REPLY

E. V. Wilms

4301-65 *Swindon Way, Winnipeg, Manitoba, Canada R3P0T8*

AND

H. Cohen

Department of Mathematics, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2

(*Received 23 November 1999*)

Professor Hagedorn points out that engineers may wish to damp out free vibrations with as few dashpots as possible.

It appears that all motion will be damped out with just one dashpot, if it is located between two points which always have a non-zero relative displacement for all the undamped mode shapes. Stephen [1] pointed this out for the special case of two degrees of freedom. (Note that it resulted in a highly unsymmetrical system.)